Math 522 Spring 2014 Final Exam

Please submit your answers on separate paper. Clearly indicate what work goes with which problem, and leave adequate white space throughout. Be sure to show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish considered – this may raise your grade. You have 2 hours, and may use calculators as well as your book and any notes or papers.

Please choose 10 of the following 11 problems to complete. If you do all 11 I will choose 10 at random to grade.

1. Use the Euclidean algorithm to find $\gcd(3185, 3549)$. Then find $\text{lcm}(3185, 3549)$.

2. Use the Euclidean algorithm to find a solution to $84x + 17y = 1$.

3. Use the Chinese Remainder Theorem to solve the system:
   $\{x \equiv 8 \pmod{12}, x \equiv 8 \pmod 9, x \equiv 14 \pmod{15}\}$.

4. Use the Lifting Theorem to solve $2x^3 + 11x^2 - 3x + 2 \equiv 0 \pmod{49}$.

5. Use Euler's Theorem to find the least nonnegative residue of $7^{111,111,111} \pmod{30}$.

6. Find the order of 5, modulo 509. Hint: 509 is prime.

7. Use Quadratic Reciprocity to determine if there are solutions to $x^2 - x - 1 \equiv 0 \pmod{23}$.

8. Calculate the Jacobi symbol $\left(\frac{44}{501}\right)$.

9. Find the continued fraction representation for $\frac{47}{1003}$.

10. Use continued fractions to solve $84x + 17y = 1$.

11. Find the (repeating) continued fraction for $\sqrt{27}$.